



**ON SOME SUBORDINATION PROPERTIES OF A NEW SĂLĂGEAN-TYPE
INTEGRAL OPERATOR**

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Abstract

In this paper, author introduce a new Sălăgean-type integral operator in the open unit disk by the term of the extended generalized Mittag-Leffler function. Based on this operator, we investigate some new differential subordination for holomorphic functions having the property of bounded turning.

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1. Introduction and preliminaries

In complex analysis, geometric function theory (GFT) is a remarkable area. It deals and studies the geometric properties of complex holomorphic (regular) functions. This theory was created by Riemann [1] in 1851, when Riemann given a considerable outcome named the "Riemann mapping theorem". The Theory of Univalent holomorphic Functions (UHFT) is the cornerstone of Geometric Function Theory (GFT). Indeed, in 1907, Koebe [1] introduced the notion of univalent function, which contributed significantly to the origin

of the Univalent holomorphic Function Theory. Let $H(\Delta)$ denote the class of regular functions in the open unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$. For $\alpha \in \mathbb{C}$ and $k \in \mathbb{N}$, let

$$H[\alpha, \kappa] = \left\{ \varphi \in H(\Delta) : \varphi(z) = \alpha + \alpha_{\kappa} z^{\kappa} + \alpha_{\kappa+1} z^{\kappa+1} + \dots \right\}$$

.Let

$$A_{\kappa} = \left\{ \varphi \in H(\Delta) : \varphi(z) = z + \alpha_{\kappa} z^{\kappa} + \alpha_{\kappa+1} z^{\kappa+1} + \dots \right\}.$$

Further, denoted by A the class of functions φ of the form $\varphi(z) = z + \sum_{\kappa=2}^{\infty} \alpha_{\kappa} z^{\kappa}$, that are normalized regular functions in Δ . The subclass of A including univalent functions denoted by s . Thus s is the class of all

normalized univalent functions in Δ . In 1913, Study [2] considered the concept of convex function as: A function $\varphi \in A$ is said to be a convex function if the image $\varphi(\Delta)$ is convex domain: that means for any $z_1, z_2 \in \varphi(\Delta)$, implies that $\xi z_1 + (1 - \xi)z_2 \in \varphi(\Delta)$ for $0 \leq \xi \leq 1$. Expressed analytically, this geometric property is equivalent to the condition $\Re\left(1 + \frac{z\varphi''(z)}{\varphi'(z)}\right) > 0, z \in \Delta$. The subclass of functions $\varphi \in A$ consisting of convex functions is denoted by CV , and $CV \in S$, [2]. Another subclass of S is the subclass $B(A)$ that includes functions $\varphi \in A$ so that

$$\Re(\varphi'(z)) > \wp, (0 \leq \wp < 1, z \in \Delta). \tag{1}$$

The functions in $B(A)$ are called functions of bounded turning functions (functions whose derivatives have positive real parts). Obviously, $B(A) \subseteq B(0) =: B$, which was first introduced and studied by MacGregor [3] in 1972. More recently, this type of subclass has been studied and generalized by various mathematicians, ([4], [5]).

The theory of differential subordination is the complex analogue formal of differential inequality on the real line, that has developed over the recent years. Valuable reference on this study is in [6] This topic has been pursued and investigated by different researchers, one may point to ([4], [7]-[10]). Recall that for $\phi, \psi \in H(\Delta)$ The function ϕ is

subordinate to ψ , written $\phi \prec \psi$, if there is a Schwarz function φ , holomorphic in Δ with $\varphi(0) = 0$ and $|\varphi(z)| < 1, z \in \Delta$ such that $\phi(z) = \psi(\varphi(z))$. In particular, if the function ψ is univalent in Δ , then $\phi \prec \psi$ if and only if $\phi(0) = \psi(0)$ and $\phi(\Delta) \subseteq \psi(\Delta)$. The following concept was given by Miller and Mocano [6]:

Let $\Phi : \square^2 \times \Delta \rightarrow \square$ and let ν be univalent in Δ . If μ is regular in Δ and satisfies the differential subordination,

$$\Phi(\mu(z), z\mu'(z), z) \prec \nu(z), \tag{2}$$

then μ is said to be a solution of the differential subordination (2). The univalent function ω is called a dominant of the solution of the differential subordination (2), or simply dominant, if $\mu \prec \omega$ for all μ satisfies (2). A dominant $\tilde{\omega}$ that satisfies $\tilde{\omega} \prec \omega$, for all dominant ω of (2), is said to be the best dominant.

The following outcomes are required in this investigation.

Lemma 1.1 [11] Let ν be a convex function, with $\nu(0) = \rho$ and let $\zeta \in \square := \square \setminus (0)$ be a complex number with $\Re(\zeta) \geq 0$. If $\mu \in H[\alpha, \kappa]$ and

$$\mu(z) + \frac{1}{\zeta} z \mu'(z) \prec \nu(z), \tag{3}$$

then $\mu(z) \prec \omega(z) \prec \nu(z)$, where

$$\omega(z) = \frac{\zeta}{\kappa z \zeta / \kappa} \int_0^z t^{((\zeta/\kappa)-1)} \nu(t) dt. \tag{4}$$

The function ω is convex in Δ and is the

best dominant.

Lemma 1.2 [12] Let $\Re(r) > 0$ and

$$\rho = \frac{\kappa^2 + |r|^2 - |\kappa^2 - r^2|}{4\kappa \Re\{r\}}. \tag{5}$$

Let v be a regular function in Δ with $v(0) = 1$ and suppose that

$$\Re\left(1 + \frac{z v''(z)}{v'(z)}\right) > -\rho. \tag{6}$$

If $\mu(z) = 1 + \mu_\kappa z^\kappa + \mu_{\kappa+1} z^{\kappa+1} + \dots$ is regular in Δ and

$$\mu(z) + \frac{1}{r} z \mu'(z) < v(z), \tag{7}$$

then $\mu(z) < \omega(z)$, where ω is a solution of the differential equation

$$\omega(z) + \frac{\kappa}{r} z \omega'(z) = v(z), \omega(0) = 1, \tag{8}$$

given by

$$\omega(z) = \frac{r}{\kappa z^{r/\kappa}} \int_0^z t^{((r/\kappa)-1)} v(t) dt. \tag{9}$$

Moreover ω is the best dominant.

In Geometric Function Theory (GFT), a variety of linear and nonlinear (differential and integral) operators have been considered. Operators are actually used to study new subclasses and their geometric properties. In 1915, Alexander [13] introduced a first integral operator on A named Alexander operator and defined as:

$$I\varphi(z) = \int_0^z \varphi(t) t^{-1} dt. \tag{10}$$

This study was followed by Libera [14], Bernardi [15] and others. In 1983, Sălăgean [16] imposed a τ -order integral

operator $I^\tau \varphi(z)$, namely Sălăgean integral operator, as follows: For a function

$\varphi \in A, \tau \in \mathbb{R}$ and $I^\tau : A \rightarrow A$, such that

$$\begin{aligned} I^0 \varphi(z) &= \varphi(z), \\ I^1 \varphi(z) &= I \varphi(z) = \int_0^z \varphi(t) t^{-1} dt, \\ &\vdots \\ I^\tau \varphi(z) &= I(I^{\tau-1} \varphi(z)) = z + \sum_{\kappa=2}^{\infty} \frac{\alpha_\kappa}{\kappa^\tau} z^\kappa. \end{aligned} \tag{11}$$

This operator (11) reduces to Alexander operator (10) if $\tau = 1$. It has attracted the attention of many researchers. One may refer to some of their contributions [17] and [18].

Special Functions (SF) are another important type of functions that are defined by the power series. They are a significant theme in mathematics dating back to the 19th century. The important role played by special functions in Geometric Function Theory (GFT), particular by the hypergeometric function in proofing the famous problem in 1984 so-called Bieberbach's conjecture [2], gives function theorists a renewed impulse to study the classes of special functions.

In 1903, Mittag-Leffler [19] imposed the following special function:

$$M_\eta(z) = \sum_{\kappa=0}^{\infty} \frac{z^\kappa}{\Gamma(\eta\kappa-1)!}, \quad (z, \eta \in \mathbb{R}, \Re\{\eta\} > 0). \tag{11}$$

This function is a generalization of the exponential function, which has numerous applications in various fields engineering and science. Furthermore, several authors have

investigated and studied Mittag-Leffler type functions. For instance, Srivastava et al. [20], Srivastava and Bansal [21] and others.

In 2017, Rahman et al. [22] introduced the extended generalized Mittag-Leffler function as:

$$M_{\eta,\gamma}^{\delta,q,a}(z;p) = \sum_{\kappa=0}^{\infty} \frac{B_p(\delta+\kappa q, a-\delta) (a)_{\kappa q}}{B(\delta, a-\delta) \Gamma(\eta\kappa-1)! \kappa!} z^{\kappa},$$

($z, \eta, \gamma, \delta, a \in \mathbb{C}, \Re\{a\} > \Re\{\delta\} > 0, \Re\{\eta\} > 0, \Re\{\gamma\} > 0$).
 ... (12)

where $B(n, m)$ is the classical Beta function and $B_p(n, m)$ is the extended Beta function as:

$$B_p(n, m) = \int_0^1 t^{n-1} (1-t)^{m-1} e^{-p/t(1-t)} dt,$$

($\Re\{\eta\} > 0, \Re\{\gamma\} > 0, \Re\{p\} > 0$).
 ... (13)

Motivated by previous works, we define a new Sălăgean-type integral operator in the open unit disk by the term of the extended generalized Mittag-Leffler functions. Based on this operator, a subclass of holomorphic functions having the property of bounded turning is introduced. Further, some results concerning an application of differential subordination for functions involved in this new subclass are discussed.

2. Imposed Operator $\Omega_{\sigma}^{\tau} \varphi(z)$

This section presents a new Sălăgean type integral operator $\Omega_{\sigma}^{\tau} \varphi(z)$ by using the extended generalized Mittag-Leffler functions $M_{\eta,\gamma}^{\delta,q,a}(z;p)$ given in (13).

Firstly, we modify the extended generalized Mittag-Leffler function $M_{\eta,\gamma}^{\delta,q,a}(z;p)$ defined in (13) as:

$$Q_{\eta,\gamma}^{\delta,q,a}(z;p) = \left[\frac{\Gamma(\gamma) B(\delta, a-\delta)}{B_p(\delta, a-\delta)} \right] z M_{\eta,\gamma}^{\delta,q,a}(z;p)$$

$$= z + \sum_{\kappa=2}^{\infty} \frac{\Gamma(\gamma) B_p(\delta+(\kappa-1)q, a-\delta) (a)_{(\kappa-1)q}}{B_p(\delta, a-\delta) \Gamma(\eta(\kappa-1)+\gamma) (\kappa-1)!} z^{\kappa}.$$

.... (14)

By employing the function $Q_{\eta,\gamma}^{\delta,q,a}(z;p)$ given in (14), we consider the following Sălăgean-type integral operator: For a function $\varphi \in \mathbf{A}, \tau \in \mathbb{C}, 0 \leq \sigma \leq 1$ and

$\Omega^{\tau} : \mathbf{A} \rightarrow \mathbf{A}$, such that

$$\Omega_{\sigma}^0 \varphi(z) = \varphi(z),$$

$$\Omega_{\sigma}^1 \varphi(z) = \frac{((\sigma/1-\sigma)+1)}{z((\sigma/1-\sigma)-1)} \int_0^z ((\sigma/1-\sigma)-1) \left(\Omega_{\eta,\gamma}^{\delta,q,a}(z;p) \varphi(z) \right) dt$$

$$= z + \sum_{\kappa=2}^{\infty} \left[\frac{\Gamma(\gamma) B_p(\delta+(\kappa-1)q, a-\delta) (a)_{(\kappa-1)q}}{B_p(\delta, a-\delta) \Gamma(\eta(\kappa-1)+\gamma) (\kappa-1)! [(\sigma+\kappa(\sigma-1))]} \right] \alpha_{\kappa} z^{\kappa},$$

⋮
 ⋮
 $\Omega_{\sigma}^{\tau} \varphi(z) = \Omega_{\sigma}(\mathbf{D}_{\sigma}^{\tau-1} \varphi(z))$

$$= z + \sum_{\kappa=2}^{\infty} \left[\frac{\Gamma(\gamma) B_p(\delta+(\kappa-1)q, a-\delta) (a)_{(\kappa-1)q}}{B_p(\delta, a-\delta) \Gamma(\eta(\kappa-1)+\gamma) (\kappa-1)! [(\sigma+\kappa(\sigma-1))]} \right]^{\tau} \alpha_{\kappa} z^{\kappa}.$$

.... (15)

Remark 2.1 Note the following special cases:

1. For $\tau = 0$, we acquire $\Omega_{\sigma}^0 \varphi(z) = \varphi(z)$,
2. For $\sigma = 0$ and $p = q = \gamma = \delta = \eta = \tau = 1$, we achieve $\Omega_0 \varphi(z) = z + \sum_{\kappa=2}^{\infty} \frac{\alpha_{\kappa}}{\kappa!} z^{\kappa}$.
3. For $\sigma = 0$ and $p = q = \gamma = \delta = \eta = 1$, we yield $\Omega_0^{\tau} \varphi(z) = z + \sum_{\kappa=2}^{\infty} \left[\frac{1}{\kappa!} \right]^{\tau} \alpha_{\kappa} z^{\kappa}$.

3. Differential Subordination for $\Re_{\tau}(\phi)$

This section is devoted to provide a subclass $\Re_{\tau}(\phi)$ of bounded turning functions that comprises a new operator $\Omega_{\sigma}^{\tau} \phi(z)$ defined in (15). Moreover, several applications of differential subordination for functions included in this considered subclass are investigated.

Definition 3.1 Let $\phi \in A$, then $\phi \in \Re_{\tau}(\phi)$ [the subclass of bounded turning functions that contains the operator (15)] if it satisfies the following inequality:

$$\Re\left(\left(\Omega_{\sigma}^{\tau} \phi(z)\right)'\right) > \phi, \quad (0 \leq \phi < 1, z \in \Delta). \tag{16}$$

Remark 3.1 For $\tau=0$, the subclass $\Re_{\tau}(\phi)$ (16) reduces to the subclass $B(\phi)$ (1).

Theorem 3.1 The set $\Re_{\tau}(\phi)$ is convex.

Proof. Let $\phi_n(z) = z + \sum_{\kappa=2}^{\infty} \alpha_{\kappa,n} z^{\kappa}$ ($n=1, 2, \dots, j$)

be in the subclass $\Re_{\tau}(\phi)$. It is sufficient to show that the function $\theta(z) = \sum_{n=2}^j c_n \phi_n(z)$ with

c_1, c_2, \dots, c_n nonnegative and $\sum_{n=2}^j c_n = 1$ is in

$\Re_{\tau}(\phi)$. Since $\theta(z) = z + \sum_{\kappa=2}^{\infty} \left(\sum_{n=2}^j c_n \alpha_{\kappa,n} \right) z^{\kappa}$, then

$$\Re\left(\left(\Omega_{\sigma}^{\tau} \theta(z)\right)'\right) > 1 + \sum_{n=2}^j c_n (\phi-1) = \phi. \quad \text{Thus, we}$$

gain the desired result.

Theorem 3.2 Let ω be convex function in Δ with $\omega(0) = 1$ and let

$$v(z) = \omega(z) + \frac{z}{\zeta+2} \omega'(z), \tag{17}$$

where ζ is a complex number with $\Re(\zeta) > -2$ and $z \in \Delta$. If $\phi \in \Re_{\tau}(\phi)$ and $F = I_{\zeta}(\phi)$, where

$$F(z) = I_{\zeta}(\phi)(z) = \frac{\zeta+2}{z^{\zeta+1}} \int_0^z t^{\zeta} \phi(t) dt, \tag{18}$$

then

$$\left(\Omega_{\sigma}^{\tau} \phi(z)\right)' \prec v(z), \tag{19}$$

implies

$$\left(\Omega_{\sigma}^{\tau} F(z)\right)' \prec \omega(z), \quad (z \in \Delta), \tag{20}$$

and this result is sharp.

Proof. From the equality (18), we obtain

$$z^{\zeta+1} F(z) = (\zeta+2) \int_0^z t^{\zeta} \phi(t) dt, \quad (\Re(\zeta) > -2, z \in \Delta), \tag{21}$$

Differentiating (21), with respect to z , we yield

$$(\zeta+1) F(z) + z F'(z) = (\zeta+2) \phi(z)$$

and

$$(\zeta+1) \Omega_{\sigma}^{\tau} F(z) + z \left(\Omega_{\sigma}^{\tau} F(z)\right)' = (\zeta+2) \Omega_{\sigma}^{\tau} \phi(z) \tag{22}$$

Differentiating (22), we deduce

$$\left(\Omega_{\sigma}^{\tau} F(z)\right)' + \frac{1}{\zeta+2} z \left(\Omega_{\sigma}^{\tau} F(z)\right)'' = \left(\Omega_{\sigma}^{\tau} \phi(z)\right)' \tag{23}$$

Utilizing the differential subordination (19) in the equation (23), we have

$$\left(\Omega_{\sigma}^{\tau} F(z)\right)' + \frac{1}{\zeta+2} z \left(\Omega_{\sigma}^{\tau} F(z)\right)'' \prec v(z). \tag{24}$$

Let

$$\rho = \frac{\kappa^2 + |r|^2 - |\kappa^2 - r^2|}{4 \kappa \Re \{r\}}. \tag{30}$$

Let v be a regular function in Δ with $v(0) = 1$ and suppose that

$$\Re \left(1 + \frac{z v''(z)}{v'(z)} \right) > -\rho. \tag{31}$$

If $\varphi \in \Re_{\tau}(\varphi)$ and $F = I_{\zeta}(\varphi)$, where F is defined in (18), then

$$\left(\Omega_{\sigma}^{\tau} \varphi(z) \right)' \prec v(z), \tag{32}$$

implies

$$\left(\Omega_{\sigma}^{\tau} F(z) \right)' \prec \omega(z), \tag{33}$$

and ω is the solution of the differential equation

$$v(z) = \omega(z) + \frac{1}{\zeta+2} z \omega'(z), \quad \omega(0)=1, \tag{34}$$

given by

$$\omega(z) = \frac{\zeta+2}{z^{\zeta+2}} \int_0^z t^{\zeta+1} v(t) dt. \tag{35}$$

Moreover ω is the best dominant.

Proof. By considering $\kappa = 1$ and $r = \zeta + 2$ in Lemma 1.2, then the result is yielded by using the similar manner of the proof of Theorem 3.2.

Remark 3.2 Considering

$$v(z) = \frac{1 - (2\varphi - 1)z}{1+z}, \quad (0 \leq \varphi < 1), \tag{36}$$

in Theorem 3.3, we achieved the following outcome.

Theorem 3.4

If $0 \leq \varphi < 1, 0 \leq \sigma \leq 1, \tau \in \mathbb{R}_0, \Re(\zeta) > -2$ and

$F = I_{\zeta}(\varphi)$ is defined in (18), then

$$I_{\zeta}(\Re_{\tau}(\varphi)) \subset \Re_{\tau}(\ell), \tag{37}$$

where $\ell = \min_{|z|=1} \Re_{\tau}(\ell) = \ell(\tau, \varphi)$ and this result is

sharp. Further

$$\ell = \ell(\tau, \varphi) = 2\varphi - 1 + 2(\zeta+2)(1-\varphi)\psi(\zeta), \tag{38}$$

where

$$\psi(\zeta) = \int_0^z \frac{t^{\zeta+2}}{1+t} dt. \tag{39}$$

Proof. If we let $v(z) = \frac{1 - (2\varphi - 1)z}{1+z}$, then v is convex and by Theorem 3.3, we deduce

$$\begin{aligned} \left(\Omega_{\sigma}^{\tau} F(z) \right)' \prec \omega(z) &= \frac{\zeta+2}{z^{\zeta+2}} \int_0^z t^{\zeta+1} \frac{1 - (2\varphi - 1)t}{1+t} dt \\ &= 2\varphi - 1 + \frac{2(\zeta+2)(1-\varphi)}{z^{\zeta+2}} \int_0^z \frac{t^{\zeta+1}}{1+t} dt \\ &= 2\varphi - 1 + \frac{2(\zeta+2)(1-\varphi)}{z^{\zeta+2}} \psi(\zeta), \end{aligned} \tag{40}$$

where $\psi(\zeta)$ is given by (39). If $\Re(\zeta) > -2$, then from the convexity of ω and the fact that $\omega(\Delta)$ is symmetric with respect to the real axis, we have

$$\begin{aligned} \Re \left\{ \left(\Omega_{\sigma}^{\tau} F(z) \right)' \right\} &\geq \min_{|z|=1} \Re \{ \omega(z) \} = \Re \{ \omega(1) \} = \ell(\tau, \varphi) \\ &= 2\varphi - 1 + 2(\zeta+2)(1-\varphi)\psi(\zeta), \end{aligned} \tag{41}$$

where $\psi(\zeta)$ is given by (39). From (40), we obtain $I_{\zeta}(\Re_{\tau}(\varphi)) \subset \Re_{\tau}(\ell)$, where ℓ is given by (28).

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